On the Metaphysics of Least Action

Benjamin T. H. Smart*[1] and Karim P. Y. Thébault†[2]

[1] *Department of Humanities*, University of Johannesburg, Johannesburg, South Africa
[2] *MCMP*, Ludwig Maximilians Universität, Munich, Germany

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Abstract

When we predict the evolution of physical systems, there are two formally equivalent, but conceptually distinct types of what we might call 'fundamental law' available to us: there are those laws philosophers are most used to talking about – instantaneous Newtonian-style laws whereby we capture the state of a system at a time, apply the laws of nature, and predict the state of the system at a later time. However, there are also, within fundamental physics at least, laws of a different type – atemporal laws that are not defined at a given time. These laws say that, for a given system, and specified initial and final conditions, the system will evolve through a sequence of instantaneous states that minimises a quantity called 'action'. In a recent paper, Stephen Barker identified what he calls 'the three degrees of modal involvement' (Barker 2013), which together exhaust the popular contemporary metaphysical accounts of laws of nature. In this paper we consider whether or not the Principle of Least Action (PLA) can be accommodated by each degree, concluding that, contra Joel Katzav (Katzav 2004; Katzav 2005), even the dispositional essentialist can be successful in this regard.

Keywords: Dispositionalism, Armstrongianism, Humeanism, Laws of Nature, Action Principles

1 Introduction

Although it is not uncommon for metaphysicians to put the fundamental laws to one side and discuss, say, causation in terms of macroscopic objects such as vases, matches and so on (Mumford and Anjum 2011), in this paper we are concerned with the metaphysics of

*email: ben@jerseyserve.com
†email: karim.thebault@gmail.com

our most fundamental physical laws. When we predict the evolution of physical systems, there are two *formally equivalent*[1], but conceptually distinct types of what we might call ‘fundamental law’ available to us: there are those laws philosophers are most used to talking about – instantaneous Newtonian-style laws whereby we can take the state of a system at a time, apply the a laws of nature, and predict the state of the system at a later time. These types of laws are often referred to as ‘equations of motion’, and occupy almost all of the metaphysical discussion of laws. However, there are also, within fundamental physics at least, laws of a different type – atemporal laws (i.e., ones that are not defined at a given time). These laws say that, for a given system, and specified initial and final conditions, the system will evolve through sequences of instantaneous states which minimize, or to be more precise *extremizes*, a quantity called action.[2] Like instantaneous laws, atemporal laws come in different forms depending upon the physical characteristics of the systems we are considering. However, unlike for instantaneous laws, there is a fundamental meta-level law underlying all atemporal nomology: the principle of least action (PLA): that *in general, for any system, and any boundary conditions, action will be extremized*.

This paper is devoted to considering the metaphysical implications of the PLA. By this we will not mean anything like a notion of fundamental teleology as has, at times, been done historically.[3] Rather, we will follow the attitude of Stöltzner and consider action principles as a providing a distinct conceptual scheme for looking at a physical problem than the equations of motion: ‘Whereas the [instantaneous laws] focus on the local aspect of the dynamics...the action principle expresses a level of higher structure’ (Stöltzner 1994, pp. 33-34) and, furthermore, systematically relates the modal structure of the actual dynamics and the possible dynamics (Stöltzner 2009). Given such an outlook, our analysis will focus upon the implications of the PLA in the context of various metaphysics of laws of nature, that is, a number of metaphysical conceptions of natural law that provide different understandings of physical and metaphysical possibility, as distinguished in terms of Barker’s ‘three degrees of modal involvement’ (Barker 2013). We can crudely characterise these as: (i) ‘powerless’ laws supervening on patterns of property instantiations; (ii) laws as higher-order facts concerning ‘necessitation-relations’ between universals[4]; and (iii) physical modality deriving entirely from ‘powerful’ first order properties.[5] We will consider what the PLA should be taken to be

[1]Here we mean mathematical equivalence between the relevant formulations of the dynamical laws within a physical theory. This equivalence can only generally be proved given further conditions. See footnote 10 for more details.

[2]This distinction with regard to laws of nature corresponds precisely to the difference between the ‘Newtonian schema’ and ‘Lagrangian schema’ discussed in (Wharton 2012). There the author argues in favour of the latter.

[3]For detailed analysis of this and other, chiefly historical, issues regarding the philosophical status and implications of PLA beyond the scope of our current discussion see (Stöltzner 1994; Stöltzner 2003; Stöltzner 2009).

[4]We leave open the possibility of a nominalist version of the second-degree – if such a view is feasible, it is beyond the scope of this paper to develop

[5]This is not necessarily an exhaustive list of conceptions of natural law, but they are certainly the most discussed in the literature.

ontologically for each of the three degrees, its modal profile, and where the PLA appears in the explanatory hierarchy. We conclude (contra (Katzav 2004)) that all three degrees of modal involvement can accommodate the PLA without undue discomfort.

2 The Lazy Plumber and The Principle of Least Effort

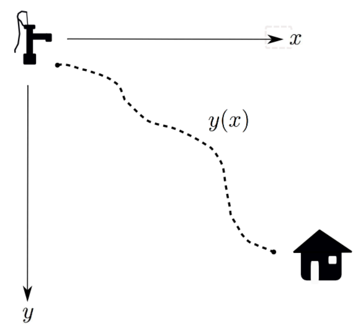
Since action principles and state spaces are rather unintuitive things, we will provide a *metaphorical* introduction to them via the *Fable of the Lazy Plumber*. Although, this fable is as an intuitive story rather than a genuine physical model, it will give the reader unfamiliar with mathematical physics a useful heuristic for the key formal concepts introduced later.

A man wants to pump water from a well at one end of a steep, smooth valley to his house at the other end using a long piece of flexible rubber tubing. Being a lazy man (who would rather spend his time reading philosophy) he decides he only wants to spend at the very most half an hour a day pumping water. Starting at his house, he rides to the well on his donkey laying the tubing along the valley as he goes in a haphazard manner, without any concern for which path will allow the water to run most easily. This initial path between the two fixed endpoints (i.e. the well and his house) proves inadequate since the energy it takes for the water to run the distance of the piping (sometimes against the gradient sometimes with it) is such that only a very small amount makes it to his house after his half hour of pumping. To remedy this, the next day he adjusts the path of the tubing slightly, and observes that evening that a little more water has come through – but still not enough for his daily household tasks. After spending several weeks adjusting the path of the pipe each day and observing that sometimes more and sometimes less water comes through, he finds a path where enough water comes out. However, by this time, maddened by the tedium of his task, he has decided that he must establish not simply a good path, but the best possible path.

Although our lazy plumber knows only a very little physics, he does know some mathematics, and rather than spending the rest of his life simply trying out every single one of the infinite number of paths, he thinks up a methodology for proving that a given path either is or is not the best. He reasons that how much effort[6] it takes for the water to get through the entire pipe (and therefore the total amount of water that makes it through after his half hour of pumping) can be represented as follows:

Figure 1.

[6]The plumbers notion of effort is, of course, not a real concept used in physics – rather we use it here to provide a intuitively helpful analogy to the fundamental physical idea of action that will be introduced in the next section.



Each point in the valley can be labeled by the horizontal distance from the well, x, and the vertical distance from the well, y [7]. A given path for the piping can then be represented in terms of a curve of the form $y(x)$.

The characteristics of each point in the valley allow him to associate it with a number indicating the amount of effort it takes for the water to pass through the pipe when laid across it. This association between numbers and points in the valley constitutes an effort function $f(x, y)$. Knowing some elementary calculus, the man realizes that for a given path the total effort to pass the length of the pipe, E, is going to be given by integrating the effort function along that path.

Each point in the valley is labelled by co-ordinates x and y. The function f(x,y) gives the 'effort' required for water to flow though a given point. The total effort required for a path $y(x)$ is given by the equation:

$$E = \int_{y(x)} f(x, y)dx$$

In order to find the best path he must find the $y(x)$ which makes E is as small as possible. Now of course, he can't just try out every single possible path since they are infinite! Rather he needs a methodology for evaluating a given path as the best simply from the knowledge that the effort will be at its minimum value for such a path. Again drawing on his knowledge of calculus (specifically the calculus of variations) he reasons that the least effort path will be the one for which very small variations in the path produce no change in E. This is because finding such an extremal path indicates that the quantity E is stationary (either maximum, minimum or an inflection) and given the physical structure of the problem only the minimum option is possible. So the problem of our lazy plumber is solved by insisting that for variations between paths infinitesimal close to his test path there is no change in the amount of effort ($\delta E = 0$), and so we have arrived at a principle of least effort: the path which extremizes the effort is the best possible path. Having thus discovered the principle of least effort our lazy plumber can then rest easy in the knowledge that he is obtaining his daily water supply in the most efficient possible manner and go back to reading his philosophy books.

[7] Assume that the path cannot loop back on itself such that some x coordinate delivers two y values

3 The Principle of Least Action

Consider two uncharged massive point particles whose motion in three dimensional space is described by Newtonian mechanics. At a given time we can represent the state of this system in terms of six configuration variables (three for each particle) and six velocity variables (again three for each particle). For a two particle system we could describe the state at a given times by a point in a 12 dimensional space. Extending this to N particles, since we still have three position and three velocities per particle, we would need a point in a 6N dimensional space to represent a moment in time. Such a state space is know as velocity-configuration space, $T\mathcal{C}$.[8]

Now, a path through this space is a sequence of possible states, and therefore represents a history of the system. Different paths (different histories) involve the system passing through more or less *energetically demanding* sequences of possible states. It is found in nature that the path that the system actually takes (that which is instantiated in reality) is the one which is in, a specific sense, least demanding. Thus it seems nature must implement a principle for choosing the most efficient path similar to that used by our lazy plumber, only the valley that must be spanned is the space of possible instantaneous states, and the paths represent entire histories of the system in question.

By convention we write the position variables $\mathbf{q}_i$, and the velocity variables $\dot{\mathbf{q}}_i$, where bold indicates a *vector* quantity, and the index i labels the number of the particle (i.e., i=1,...N). Thus a single instantaneous state of the two particle system is specified by the variables $\mathbf{q}_1, \mathbf{q}_2, \dot{\mathbf{q}}_1, \dot{\mathbf{q}}_2$.

For a given physical system there is a special function particular to it called the Lagrangian, L, which associates each point in velocity-configuration space with a number. The Lagrangian is thus a *functional* of all the degrees of freedom of a classical system, $L(\mathbf{q}_i, \dot{\mathbf{q}}_i)$. Formally, this is a *map* from the velocity-configuration space to the real numbers:

$$L(\mathbf{q}_i, \dot{\mathbf{q}}_i) : T\mathcal{C} \to \mathbb{R} \tag{1}$$

The functional form of the Lagrangian is fixed by the physical characteristics of the system we are dealing with, or, in other words, the properties and forces that are involved. Thus, for example, a Lagrangian for a system with charged massive particles will include a terms relating to electrostatic interaction and terms relating to gravitational interactions. For this reason (Katzav 2004, p.208), reasonably, takes the Lagrangian to be a ‘function of all the...intrinsic properties ascribed to the objects in the system by classical mechanics’

In general, the explicit form of L is given simply by the kinetic energy of the system T minus the potential energy V.[9] For our two particle Newtonian system the Lagrangian can

[8]This notation derives from the formal character of this space as the *co-tangent bundle* to the confirmation space.

[9]The Lagrangian formulation of the General Theory of Relativity is an important exception to this.

be written using the two position vectors, $\mathbf{q}_1$ and $\mathbf{q}_2$, as:

$$\begin{aligned} L &= T - V \\ &= \frac{1}{2}m_1\left[\frac{d\mathbf{q}_1}{dt}\right]^2 + \frac{1}{2}m_2\left[\frac{d\mathbf{q}_2}{dt}\right]^2 - G\frac{m_1 m_2}{|\mathbf{q}_1 - \mathbf{q}_2|} \end{aligned}$$

The action, I, of a physical system is then defined between two points in velocity-configuration space for any given path, γ, between those two points. A direct physical interpretation of action is not generally given in physical theory, but its SI units of joules-seconds indicate a close connection with both energy and time (and justifies our analogy with the plumbers concept of effort). The action can be explicitly calculated by the integral of the Lagrangian with respect to time along a path, γ.

$$I[\gamma] = \int_\gamma L dt$$

The principle of least action (PLA) is then that, the actual path taken by a real physical system in velocity-configuration space is that which extremizes the action (where the variation of action upon infinitesimal variation in the path, with fixed end-points, is equal to zero – $\delta I = 0$). Significantly, one can derive the equations of motion for a system using purely its Lagrangian and the PLA, and vice versa, and thus it provides a formally equivalent, yet conceptually distinct nomological schema to that given by the instantaneous laws.[10]

Figure 2.

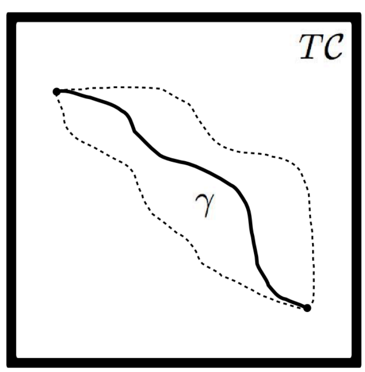


Figure 2. shows the Velocity-Configuration space, $T\mathcal{C}$, of a four-dimensional world, where each point represents the state of the world (including velocities, mass, charge etc) at a particular moment. The dotted lines represent possible paths that could have been taken to get from the first point to the second (sets of possible states the world could have taken en

[10]More precisely, the PLA allows us to derive the Euler-Lagrange equations which in turn provides us with the equations of motion for any given Lagrangian. There are some rather technical subtleties which mean that, strictly speaking, full mathematical equivalence with the instantaneous laws does not obtain. Rather, given these issues, it is only under the addition of certain formal sufficiency conditions that we can establish a rigorous proof of equivalence. See (Brechtken-Manderscheid 1991, §6).

route to the second point). The path of least action – which will always be the actual path – is shown in black.

The PLA and its formulation in terms of the system's Lagrangian is not restricted to Newtonian mechanics. It is found in all classical theories of mechanics including electromagnetism, statistical mechanics, special relativity and general relativity. Moreover, through the path integral formalism, essentially the same structure is also utilised within quantum theories also. So quantum field theory and even string theory in fact manifest a quantum version of the PLA.[11] the appropriate classical limit it is the correct expression for the quantum amplitude of a path. So in an important sense, quantum mechanics, quantum field theory and even string theory manifest a quantum version PLA. Thus, one has a solid physical basis to consider the PLA to be a universal principle.

This being so, we hope it is apparent why we feel it to be crucially important that any viable theory of laws provides an adequate metaphysical account of the principle, and where it should stand in our ontology

4 The Metaphysical Status of the PLA and Potential Problems

Following Mumford (2004), we take there to be a distinction between laws as descriptions of how physical systems evolve, and laws *in* nature. The former, which we shall refer to as 'Type-A' laws, are true statements that satisfy a particular set of criteria, but play no genuine governing role (how could they?); the latter, which we refer to as 'Type-B' laws, on the other hand, are an aspect of *being* - they are instantiated by worlds.

The intention of this paper is not to adjudicate between the three degrees of modal involvement outlined in the following section, but to analyse how the PLA might be accommodated as a law according the standards of each. We shall see that the first-degreer is committed only to Type-A laws. She does not require laws to govern, as laws are simply a special kind of 'truth' describing patterns of property instantiation. The ontological commitments of the second-degreer are somewhat stronger. The PLA, if it is to be a law, must govern the evolution of physical systems. The third-degreers are somewhat divided in their explication of what it is to be a law of nature. Their primary metaphysical claim is that physical modality comes exclusively from the dispositional nature of natural properties. Mumford (Mumford 2004) claims this should lead to a denial of laws being *in* nature. For Mumford, were the PLA to be a law it would fall under Type-A. There is nothing inconsistent, on the other hand, in mirroring Mumford's commitment to physical modality stemming purely from the intrinsic

[11] We can, for instance, consider Feynman's second postulate in his original derivation of the path integral formulation (Feynman 1948) (which partially derives its origin from remarks due to (Dirac 1933)). The postulate is that the function $e^{\frac{i}{\hbar}\int_\gamma L dt}$ will be proportional to the quantum amplitude of a classical path γ. The nature of the exponential functional form means that the amplitude will peak upon the path of the least action path (i.e. when $\delta I = 0$). It is this feature that allows us to recover the classical theory in the limit of $\hbar \to 0$. Thus, the PLA is still, in a certain sense, fundamental within the quantum realm.

nature of natural properties, yet taking laws to be Type-B phenomena that supervene upon the nature of these properties.

We will say much more on these metaphysics of laws in section 5, but one can already see that the metaphysical desiderata for laws of nature differ depending on which metaphysic one endorses; that said, Joel Katzav presents two specific problems he believes all those wishing the PLA to be a law must face.

The Many Possible Paths Problem (MPPP)

Katzav makes the non-trivial assumption that the PLA presupposes the contingency of the equations of motion within physical systems. The principle, he claims, entails that 'the action of any given physical system could have taken various values, and thus that any such system could have been correctly described by different equations of motion, even once the objects that comprise it, along with their intrinsic properties and initial distribution, are determined.' (Katzav 2004, p.210). The thought is that there are a number of ways a physical system might evolve from one state to another, represented by myriad possible values of action. Any metaphysical theory incompatible with this claim, argues Katzav, is incompatible with the PLA being a law of nature. We remain sceptical as to whether this is a genuine worry, or indeed a desirable consequence of our 'best' metaphysical theory, but we shall consider the contingency or necessity of the paths followed nonetheless.

Explanatory Force

It is generally agreed that laws must have explanatory force. What the conditions are for having explanatory force is of course hotly disputed, and often an account of laws is rejected because, or at least partly because the so called laws, as conceived by their opponent, have no explanatory power.[12] *If* the PLA is to be law it must have explanatory power [13] – and a great deal of explanatory power at that. It is, after all, a principle from which (in combination with the system's Lagrangian) all the equations of motion can be derived. In the following sections we do not presuppose any conditions for explanatory value independent of the metaphysical view – rather, we shall consider the PLA's explanatory force in the context of the metaphysical position being discussed; that is, if x requires property p for x to have explanatory power under the conditions specified by those holding metaphysical theory M, and the PLA has property p, then in the context of M the PLA has explanatory power.

There is then the further question of where in the explanatory chain the PLA sits. Given the universality of the PLA, and the fact that the many distinct equations of motion can be derived from it (along with the system's Lagrangian), we might expect the PLA to be the ultimate explanans. Katzav (Katzav 2004) even goes so far as to say that we can rule out any metaphysical position that appeals to anything other than, or in addition to the PLA,

[12] Armstrong, for example, rejects Humeanism about laws partly for this reason.

[13] Note again that the PLA might not be considered a law, and hence would not require explanatory value.

to explain the dispositions of things.[14]

In the following sections we hope to show how the proponents of each of the three degrees of modal involvement should view the PLA, whether or not these accounts successfully deal with the MPPP, and to what extent the PLA has explanatory force for each respective position.

5 Three Degrees of Modal Involvement and the PLA

5.1 The First Degree of Modal Involvement

'The first degree [of modal involvement] is that facts of physical modality are determined by, or supervene upon, features of the world or worlds *in totality*.' (Barker 2013, p.606). This view is exemplified by David Lewis's Humean Supervenience, which will be the focus of our discussion.

Take the Humean (henceforth the 'Lewisian first-degreer) account of laws to be the view that laws are mere regularities, expressed by the universal quantifications that form part of a best system of law-statements (Lewis 1986). Laws, according to this view, are contingent truths, where contingent truths are true in virtue of patterns of fundamental property instantiations and the fundamental relations between these instantiations, and in virtue of these alone; this is the consequence of endorsing Humean Supervenience – as Bigelow puts it, 'truth supervenes on being' (Bigelow 1988, p.132).

Not only do the fundamental laws or axioms of the best system count as laws, but so do any 'regularity' propositions that can be derived from these axioms. *Prima facie* this is desirable consequence, as many of the statements we accept as law-statements do not express the basic laws, but something less fundamental. The statement 'all electrons have charge $-e$', for example, does not seem to express one of the fundamental laws in our world (since it enters our current fundamental theories merely as a measured parameter), but it is widely accepted as a law of nature nonetheless.

So where does this leave the PLA? In the actual world, the natural property 'electron' is always co-instantiated with the natural property 'charge $-e$'; in other words, in the actual world the members of the set of electrons are all members of the set of things with charge $-e$. According to our best physical theories the proposition 'all electrons have charge $-e$' expresses a law, but not a fundamental one, and so according to this view it must be derivable from the fundamental axioms of the best-systems account. Suppose the fundamental equations of motion are those given by the theories of electromagnetism and Newtonian mechanics. These equations are, it turns out, mathematically equivalent to the PLA (applied to the relevant Lagrangian), so in knowing the equations of motion we can derive the PLA. For the Lewisian first-degreer, the PLA is thus a law.

[14]We take it that Katzav must be using 'disposition', here, as if reducing dispositional properties to counterfactuals was a live option. This is not the view endorsed by those committed to dispositional essentialism.

But of course the equations of motion are also derivable from just the PLA and the system's Lagrangian, so we have no obvious reason to think the equations of motion are fundamental. The equations of motion are distinct from one another – the Newtonian law of gravity makes no reference to charge, or magnetic fields. The PLA, on the other hand, is not like this. It is a single principle that encompasses all motion within a physical system. It would make more sense, it seems, to take the PLA to sit at the top of our best system; that is, to take the PLA to be the law with the best combination of simplicity and strength. The equations of motion thus retain their status as laws of nature (as they are derivable from the PLA), but lose their 'most fundamental' status.

Despite the apparent oddness of this atemporal law, there is a nice parsimony to their being a single law from which all others can be derived.

The Lewisian first-degreer, then, will have the following to say about the PLA:

1. It is our most fundamental law, from which all others can be derived.

2. As a law, it is a mere regularity, expressed by a universal quantification forming part of a best system of law-statements

3. The properties the law refers to are categorical – and in being just primitively perfectly natural sets of particulars, they have no intrinsically powerful nature. [15]

Many Possible Paths and The First Degree of Modal Involvement

For the first-degreer, no problem is posed by the MPPP, as according to Humean Supervenience, the PLA is *contingent* – for her, the PLA is a law in virtue of the path the physical system follows being that which extremises action, and not vice versa. So as Katzav requires, 'the action of any given physical system could [take] various values, and thus... any such system could [be] correctly described by different equations of motion' (Katzav 2004, p.210).

The Explanatory Problem and The First Degree of Modal Involvement

For the first-degreer, ontologically speaking, laws are nothing more than regularities. The second and third-degreers (the 'necessitarians') say this is problematic because there would be no good explanation for why the natural uniformities we observed every day have held, and we have no good reason to suppose they shall continue to do so. To see where the first-degreer believes explanatory power might lie, consider the following example provided in (Smart 2012, p.321-325): the results of the exit poll come through, and Obama has 52% of the vote. The exit poll provides a roughly random sample of the total number of voters, and it is reasonable to suppose that results of the polls will be indicative of the election results.

[15] Note that the view that laws supervene upon patterns of property instantiation, and nothing else, is perfectly consistent with their being powers or natural necessitation relations in nature – but any metaphysic involving such things would not be a first-degree view.

Sure enough, Obama gets roughly 52% of the vote. So what best explains the results of the poll? The answer, surely, is that 52% of the total population voted for Obama. Similarly, a phenomenon such as 'all spherical balls with mass are drawn to all other objects with mass' might be explained by the higher order regularity, and moreover the *law*, that 'all *objects* with mass are drawn to all other objects with mass', and so on. Higher-order regularities can, it seems, have explanatory value. One might object that we need to know about Obama's policies and the needs of the voter to explain the polls – there is a reason they voted the way they did, and the way the total population voted doesn't give us this, but the first-degreer does not need to give a reason for his regularities. There are just the patterns, and all truths supervene on these patterns. Ultimately, explanation has an epistemic aspect. We offer explanations to other people as a way of rendering a phenomenon intelligible, and it is clear that the more general generalisations we offer make less general ones intelligible. If there is a problem it must involve some metaphysically robust notion of explanation, but as we are judging the first-degreer by her own metaphysical standards, this cannot be an issue. So does the PLA have explanatory value? We claim that, *from the first-degreer's perspective*, it does. It is the highest-order regularity there is – the regularity from which all the other, lower order law-statements logically follow, and arguably thus the regularity with the most explanatory value.

5.2 The Second Degree of Modal Involvement

> The second degree of physical modal involvement is that facts of physical modality are not metaphysically determined by... patterns in the mosaic of fact, but are determined at a more fine grained level. On the second-degree view, what does the fixing is, primarily, a second-order relation holding between natural properties. (Barker 2013, p.606)

One would think that for the second-degreer, *that* some quantity is an extremum implies that, if it is actual, its actuality is not an accident. Moreover, that something is not an accident justifies appealing to it in explanation (Katzav 2004). The implication is that the PLA is not merely a regularity, but some worldly fact governing the physical system's evolution.

Modality of the second-degree, we take it, is motivated by at least two conceptual assumptions: first, that '*no facts about particulars can be the truthmakers for statements expressing [these] laws...*' (Tooley 1987, p.71); and second that the spatiotemporal distribution of properties should be determined by governing laws, rather than laws supervening on property instances, or on the intrinsic powerful properties instantiated by particulars. The second-degreers conclude that laws are higher-order facts concerning 'necessitation relations' between universals. For simplicity we focus on David Armstrong's formulation of second-degree modality.[16].

[16]Similar versions of the second-degree view were espoused by(Tooley 1987) and (Dretske 1977)

Armstrongians make the following claims:

1. That all natural properties are universals (with identities independent of their causal roles);
2. Immanent realism about universals; that is, one and the same universal is wholly present in each instantiation of it, but it exists only in its instances. (The universal 'charge' therefore exists only where there is an instance of charge, as opposed to existing transcendentally.);
3. That there is a hierarchy of universals. Examples of first order universals would be 'red' and 'mass $1kg$', but there are also first order relations (second-order universals) between first-order universals, and so on;
4. There can be natural necessitation relations[17] N that hold between two (or more) first-order universals, denoted $N(F, G)$. It is metaphysically necessary that if $N(F, G)$ then all Fs are Gs, but $N(F, G)$ is a contingent matter of fact– for the Armstrongian, $N(F, G)$ denotes a law of nature.

Although N, F and G are all universals whose causal/nomological roles are contingent (as $N(F, G)$ holds only contingently in any world), in all possible worlds in which '$N(F, G)$ and x is F' is true, 'x is also G', thus there are thus necessary connections between distinct existences according to this view. [18] The fact that action is extremised requires some metaphysically substantial explanation for any necessitarian – given that for the second-degreer, the necessary connection between 'x is F' and 'x is G' in world w is fixed by N(F,G) being true of w, the ultimate explanatia for non-accidental regularities must be these higher-order facts. There is of course the issue of what $N(F, G)$ would look like in the case of the PLA. Sometimes identifying the universals involved in natural laws looks simple enough: if one believes all protons have mass $1.67262178 \times 10^{-27}kg$, then $N(\text{Proton, Mass } 1.67262178 \times 10^{-27}kg)$ is the obvious nomic explanation. Things are not as clear with the PLA. In principle, though, we see no reason why the second-degreer cannot claim it to take the N(F,G) form. If, for example, we allow being a physical system to be a universal, and quantities of action through velocity configuration space to be universals, the PLA could take a form resembling the following:

$$Arm(PLA) : N(\text{Physical system, Extremised action})$$

[17]The relation itself is sui generis.

[18]It has been argued (Bird 2005) that there is a serious problem with the natural necessitation relations between universals view, in that Armstrong has to account for the necessitation of 'all Fs are Gs' by $N(F, G)$ – he would, Bird argues, have to posit a further necessitation relation linking $N(F, G)$ and 'all Fs are Gs', but again this would require a similar explanation, generating a regress. As has been shown in (Barker and Smart 2012), firstly this objection is equally applicable to Bird's own dispositional monist account, and secondly, so long as Armstrong is willing to accept that the kind of necessitation implicit in his view is brute, there is no regress. We shall not, therefore, rule Armstrong's view out tout court.

This higher-order fact, the Armstrongian would claim, governs physical systems such that in all worlds in which this fact holds, the property of being a physical system is co-instantiated with the property of extremised action. But if Arm(PLA) is providing the 'oomph', where does this leave the equations of motion?

Suppose two massive objects attract one another. We would have two explanations for this: an instantaneous 'equations of motion' explanation, let us call this $Arm(Gravity)$, and the atemporal PLA explanation, $Arm(PLA)$ – both expressible in $N(F, G)$ form:

$$\begin{aligned} Arm(Gravity) &\quad : N(\text{Massive Objects, Gravitational Force}) \\ Arm(PLA) &\quad : N(\text{Physical system, Extremised action}) \end{aligned}$$

Both seem to be higher order facts about the world, and both are supposed to govern the way the massive objects behave, yet they appear to be ontologically distinct; that is to say, Arm(Gravity) is not identical to Arm(PLA). If we allow the metaphysical interpretations of both the equations of motion and the PLA to be higher-order facts of the form $N(F, G)$, then the universe is rife with nomological overdetermination – although we admit that this does not necessarily render the position implausible, it is qualitatively unparsimonious, and should be avoided if at all possible. There are a number of possible solutions to consider:

First, we might consider $Arm(Gravity)$ and $Arm(PLA)$ not to be distinct existences after all, and that $Arm(Gravity)$ is in fact a determinate of the determinable $Arm(PLA)$. On closer inspection, though, this cannot be right. It may be true that the equation of motion corresponding to $Arm(Gravity)$ must be true of our world for the PLA to hold, but it cannot be a determinate of $Arm(PLA)$, as $Arm(PLA)$ is an atemporal, not an instantaneous principle.

Second, one might endorse a 'dual aspect' theory [19], whereby $Arm(Gravity)$ and $Arm(PLA)$ are just two sides of the same coin. We might say that explanations in terms of $Arm(Gravity)$and those in terms of $Arm(PLA)$ refer to the same entity: the quality of the actual world governing the evolution of its physical systems; and which formulation we should use to describe the explanation is merely context-dependent. However, the mathematical formalism of $Arm(Gravity)$ is not equivalent to the mathematical formalism of $Arm(PLA)$ plus the system's Lagrangian, as the formalism of $Arm(PLA)$ contains far more information than that of $Arm(Gravity)$. Unlike with the mathematical formalism of the PLA plus the Lagrangian, One cannot, for example, derive the laws of electromagnetism from Newton's law of gravitation. The dual aspect option also fails.

A third, *prima facie* more promising response, is to abandon $Arm(PLA)$, and take the PLA to be a Type B law supervening on the complete set of 'instantaneous' second degree laws. On this view, the atemporal nomic properties of worlds supervene upon its instantaneous nomic properties – this, of course, solves the overdetermination problem. Perhaps the second-degreer would be happy granting that this Type B version of the PLA is indeed a law, but not one of the N(F,G) variety. So far as these authors are concerned, though,

[19] See (Chalmers 2007) for the analogue in the physicalism debate.

for the second-degreer, laws are higher-order facts denoting necessitation relations between particular universals, and the concept described above does not satisfy these conditions.

Finally, we could deny governing law-status to one of these higher-order facts. If we choose to reject $Arm(PLA)$, then the one principle that holds universally within all our best physical theories turns out not to be a law of nature; this is at best counterintuitive. The same, of course, holds for $Arm(Gravity)$. We see no other plausible options, however, and given that the first three attempts (with, we admit, a certain amount of sympathy towards option three) ultimately fail, the second-degreer is forced either into admitting excessive nomic overdetermination, or denying law-status to one of the above.

When Katzav criticises dispositional essentialism (a third-degree view), he does so partly because he feels the dispositional essentialist attributes explanatory value to the dispositions of things, when the explanation of causal interactions should be exhausted by the PLA; that is, the PLA provides a metaphysically substantial explanation of the dispositions of particulars. (Katzav 2004, p.215) Katzav would thus no doubt welcome the conclusion reached above, as although $Arm(Gravity)$ is not a third degree state of affairs, the same objection would be raised against the second-degreer who wished to keep the 'infimic' laws. Katzav would be happy to dispose of $Arm(Gravity)$ altogether, leaving $Arm(PLA)$ to do all the work. If physical systems follow extremised paths, the second-degreer should take this to be non-accidental and in need of explanation, and for the Armstrongian, the extrinsically governing $Arm(PLA)$ is the best option. This might be counterintuitive, as we tend to use the (instantaneous) laws mathematically expressed by the equations of motion more regularly in explanation than the action principle formalisms, but this should not affect our metaphysical conclusions.

The Many Possible Paths Problem and the Second Degree of Modal Involvement

The MPPP is no more problematic for the second-degreer than it is for the first-degreer. The fact, $N(F, G)$, holds contingently at any world, and thus so does the type of necessitation it affords. In every possible world where $N(F, G)$ is true, 'all Fs are Gs' is true, but there are many more possible worlds where $N(F, G)$ is false. Accordingly, where $Arm(PLA)$ holds the path of extremised action is necessitated – but the necessitation itself is contingent. For the second-degreer, then, there are many metaphysically possible paths the physical system can follow through velocity-configuration space.

The Explanatory Problem and the Second Degree of Modal Involvement

There are no obvious problems concerning explanation with this view, either. For the second-degreer, laws govern the motion of the particulars physical systems, ensuring that the system as a whole follows the path of extremised action. The theory does not posit anything in addition to the natural necessitation relations between universals to explain the evolution of the physical system, and so the objection Katzav will raise against the third-degreer (see below) holds no weight.

5.3 The Third Degree of Modal Involvement

> What third-degreers promise is a vision of properties with inherent natures that in and of themselves determine the modal roles of properties. Properties are not merely necessarily linked to modal roles, ...but essentially, by their very natures, incorporating or determining modal roles or powers.(Barker 2013, p.621)

There are a number of variations of third degree views, all of which claim that at least some properties have dispositional character. Some argue that there are both categorical qualities, conferring no causal powers but grounding the structural aspects of the world, and dispositional properties providing the 'oomph' required for causal interactions to occur. [20] Our discussion of the third-degree of modal involvement will focus on the third of these options, dispositional monism, and in particular the version espoused by Alexander Bird in his (2007). Although some of the comments made here are specific to the monism views, the relevant conclusions we draw can be generalised to all third degree positions.

As we have seen, the kernel of any metaphysics of laws is the nature of the properties and relations it advocates. The third-degreer's dispositional properties have their identity fixed by the stimulus/manifestation-relations (SM-relations) they bear to one another. The causal relations between properties are thus metaphysically necessary – if F is the stimulus condition for G in the actual world, F is the stimulus condition for G in all possible worlds where F and G can be instantiated. Broadly speaking they hold (DE_p), '(that) at least some sparse, fundamental properties have dispositional essences' (Bird 2007a, p.45), where any object that possesses the dispositional essence of some potency[21], P, is disposed to manifest the corresponding disposition M under stimulus conditions S, in any possible world:

$$(DE_p) : \Box(Px \rightarrow D_{(S,M)}x)$$

Bird shows that from (DE_p) and the conditional analysis of dispositions as a necessary equivalence[22] that we can derive (I) $\Box(Px \rightarrow Sx\Box \rightarrow Mx)$, and furthermore that when we 'consider any world w and any case where some x in w possesses the potency P, where x acquires the stimulus S (that is, (II) (Px & Sx), we can ultimately derive a universal generalization of the form $\forall x((Px\&Sx) \rightarrow Mx)$.[23] The laws of motion, we take it, will be derived from the third-degreer's natural properties in this way.[24]

[20]See (Ellis et al. 2009, 93-115) for a detailed explication of this view; some take properties to have both a qualitative and a dispositional character (Heil 2003; Heil 2012); and some take properties to be 'pure powers' (Bird 2007a) (Mumford 2004).

[21]Bird tells us that 'potency' can be read as interchangeable with 'dispositional property' (Bird 2007a, p.45).

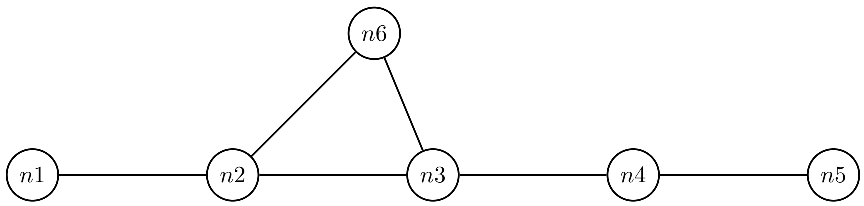
[22]$\Box(D_{(S,M)}x \leftrightarrow Sx\Box \rightarrow Mx)$

[23]From (I) and (II) we have (III) Mx, discharging (II) we have (IV) $(Px\&Sx) \rightarrow Mx$, and since x is arbitrary we may generalize $Ax((Px\&Sx) \rightarrow Mx)$ (Bird 2007a, p.46)

[24]There is discussion concerning how one should deal with cases of prevention (or finks and antidotes) – apparent exceptions to this rule, but these are of little concern for our purposes. It is also worth noting that some dispositional essentialists, notably (Mumford 2004), deny that there are laws *in* nature, that is,

The dispositional monists (unlike those third-degreers who hold a 'mixed view' of both categorical and dispositional properties) take all properties to provide 'oomph' – furthermore, they take the identities of these properties to be fixed relationally, insofar as the first-order natural properties have their identities fixed by the relations they stand in to their stimuli and manifestations. These stimuli and manifestations are natural properties in their own right, and so these, too, have their identities fixed by the stimulus-manifestation relations (SM-relations) they stand in to other natural properties. On the face of it a regress arises here, as every natural property has its identity fixed by another natural property, but it need not be [25].

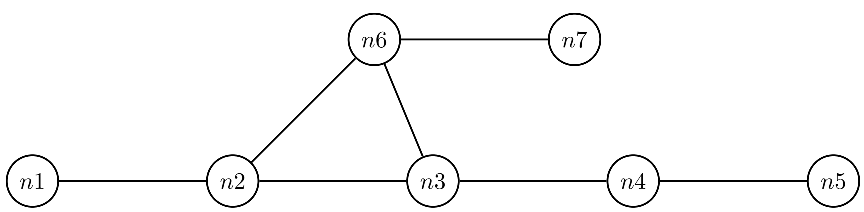
Figure 3.



Take the graph above to represent the natural properties (the nodes) and the relations fixing their identities. Aside from the graph with just a single vertex, the graph in figure 3 is the simplest non-trivial asymmetric structure that can fix the identity of properties (Bird 2007a, p.140), but it is possible to include loops and digraphs to add further asymmetries [26].

Suppose figure 3 represents the property network of the actual world, w, with only 6 fundamental properties, and where $n4$ is the natural property, 'charge $-e$'.[27] Inhabitants of w would identify 'charge $-e$' primarily with its SM-relations to $n3$ and $n5$, but more accurately its identity is fixed by its place in the entire network. Indeed, any world with 'charge' would have to be one with a property structure identical to that of w.

Figure 4.



extrinsically governing laws like those Armstrong postulates, but in endorsing Mumford's position in this respect one is not refusing intrinsically governing laws. Further discussion of this matter, however, is beyond the scope of this paper.

[25]This issue is not directly relevant to this paper, but see (Barker and Smart 2012); (Bird 2007b) for details

[26]The directedness graph demonstrates where property A can be the stimulus for property B, but not vice versa. It seems there needs to be some directedness when using graph theory, as a representation of the relations between properties and their manifestations are, in most cases, irreversible.

[27] Where the unit of charge e, is approximately equal to $1.6 \times 10^{?19}$ Coulombs

Now suppose there is a world, $w1$, with the graph structure outlined in Figure 4. Here we see an asymmetric graph structure very similar to Figure 3, but with an additional natural property, $n7$. It is possible for $n7$ never to manifest (suppose that whenever its stimulus condition is met, some other event prevents it from manifesting). Arguably, $w1$ could *appear* identical to w to my counterpart there, but it would share none of the same properties. As a property's identity is fixed by its place in the entire network, $n4$ in $w1$ would not be 'charge', but, say, 'schmarge'. Whether or not there could be possible worlds with networks different to that of the actual world is a matter beyond the scope of this paper, but this is at least a matter up for debate.

In light of the above, we take the third degree of physical modality to imply the following conception of the PLA:

The Lagrangian is fixed by the properties it is physically possible to be instantiated in that world. Each point in velocity-configuration space represents an instantaneous pattern of property instantiations, and the path (as shown in Figure 2.) represents the actual evolution of the system through various states.

For the dispositional monist, the structure of these property networks denote the nature of physical systems' natural properties, and as such how these systems evolve through velocity-configuration space – the PLA must consequently supervene upon these structures. There is no reason to think the PLA cannot be multiply realisable, of course (insofar as different graph structures in different possible worlds could give rise to physical systems following extremised paths), but this should not be surprising. After all, even in the actual world we have seen the PLA to be applicable to many different physical theories (Newtonian Mechanics, General Relativity...). For other third degree views – views purporting both dispositional and categorical properties, there is no such graph. However, a similar account of the PLA must be provided, as *all* third degree views take the evolution of physical systems to depend upon the dispositional properties instantiated by the system's particulars.

The Explanatory Problem and the Third Degree of Modal Involvement

Katzav would argue that third-degreers, like everyone else, must be able to appeal to the PLA in explanation. His reasoning is that if such a principle holds, that is, if it is true that the path of extremised action *is* followed, it would be irrational to consider this to be an accident (and the fact that it is not an accident enables us to appeal to it in explanation). Katzav does not make it clear whether he is writing about the PLA as a law of Type-A or one of Type-B, but on the face of it the explanandum must be Type-A, that is, the truth of the proposition describing the minimised path through state space, and the explanands a Type B-law, that is, a law *in* nature (some aspect of 'being') that somehow determines the evolution of the physical system. Katzav's worry seems to be that if the explanatory work is left to dispositions, then the path of least action being followed is accidental – the dispositions just happen to 'add up' to the PLA. If all the work is done by the intrinsic powerful properties of particulars, the path of least action being followed becomes an implausible cosmic coincidence. For Katzav, it seems, the third-degreer is misguided, and the metaphysician should embrace the second degree of modal involvement.

But one must question in what sense the path of least action being followed really is an accident for the third-degreer. For them, physical systems do not evolve according to least action principles by accident, but precisely because of the properties determining every causal interaction. Third-degreers take the properties to be basic, with their natures fixed by their place in a network of other natural properties (at least in the case of the dispositional monist), metaphysically necessitating the path of least action being followed.

Compare the second and third degrees of modal involvement. For the second-degreer, a law has to be a natural necessitation relation between two universals - this meant it could not be the case that both the atemporal $Arm(PLA)$ and instantaneous laws (like $Arm(Gravity)$) are Type-B without introducing an unacceptable level of nomic overdetermination. After rejecting the plausibility of our 'dual aspect theory', 'determinate-of-determinable', and supervenience responses for the second degree, however, we concluded in favour of dropping Type-B instantaneous laws altogether, in favour of allowing the N(F,G) interpretation of the PLA to do all the work. The same does not have to be done for the third-degreer, however. The dual aspect and determinate-of-determinable attempts have to be rejected on similar grounds, but the relationship between the fundamental properties and the PLA looks far more like one of supervenience [28]. After all, although the PLA might well be multiply-realisable with different property structures, the action of a PLA world cannot be anything other than extremised without a change in the structure of its property network.

In short, the third degree account of the PLA must be one of supervenience upon the nature of fundamental property-networks. We do not deny that the third-degree account of the PLA can be appealed to in explanation as a genuine, Type-B law, but we must remember what grounds it. Katzav assumes the PLA must ground the behaviour of particulars, but it is of the essence of the third-degree that the opposite is true, and there is nothing incoherent about this. Ultimately, to ask why the PLA holds for the third-degreer is to ask why the property 'mass' exists, as for 'mass' to exist, a structure that entails the PLA must be in place, and this is not a question she should be compelled to answer.

The Many Possible Paths Problem and The Third Degree of Modal Involvement

Given that physical modality comes directly from the inherent nature of the natural properties, the causal relations between properties are metaphysically necessary. Despite this, it should not be taken for granted that the PLA is a law in all possible worlds. The PLA holds with metaphysical necessity only in worlds where it supervenes upon the properties that can be instantiated there. The third degree view entails that if F is the stimulus for G then F is the stimulus for G in all possible worlds in which F can be instantiated, but it

[28] Note that a governing PLA does seem to supervene on the instantaneous laws of the second-degreer, too, which explains the initial draw to the view. However, the governing law supervening on the instantaneous laws does not take the $N(F, G)$ form essential to the second-degreer's ontology, so it could not be considered a law.

does not entail that it must be possible for F to be instantiated in all possible worlds. It is not implausible that there are worlds that do not share any of our properties, but have their own property networks – networks metaphysically necessitating that the PLA is *not* a law of nature. If there are such worlds, the PLA holds, much like in the second-degree view, as a matter of contingent necessitation[29]. Nevertheless, it remains true that there are no worlds with the same velocity-configuration space, and the same initial and final states of the system as represented by the same points in that state space, in which the paths followed differ in any respect – there is only one metaphysically possible path.

We believe, however, that Alexander Bird has successfully dealt with this objection. Bird claims that Katzav implicitly assumes that logical possibility entails metaphysical possibility, and further that the PLA only presupposes the *logical* contingency of the path (Bird 2007a, p.214). There is thus no reason to think the third-degree of modal involvement fails to satisfy the 'contingent path' requirement.

6 Concluding Remarks

For any state space, there are many possible curves between any two points, so a physical system can in principle evolve in many different ways and end up in the same final state. And yet in our world (presuming it is classical) the path of least action is always followed. It seems right to say this is no accident, and it also seems right to say that if the PLA is the reason the path followed is always that which minimizes action, it is something we can appeal to in explanation. In these respects we can agree with Katzav.

It is our contention that the first-degree of modal involvement satisfies all the criteria set forward by Katzav. Humean Supervenience, a principle that modern-day first-degreers tend to endorse, dictates that laws supervene upon a mosaic of property instantiations. When the mosaic is such that action is extremised, this atemporal regularity must be a law of nature. Furthermore, the equations of motion are laws just because they are derivable from this fundamental regularity. The second and third-degreers will complain that the laws, for the first-degreer, are accidental. But the first-degreers *do* have an adequate means of judging between accidental and non-accidental uniformities, and the PLA is non-accidental when subjected to these tests. We should judge the first-degree of modal involvement by its own standards, not by the intuitions of the necessitarians, and as the PLA comes out as a non-accidental and contingent regularity that is part of our best system of law-statements, it is undoubtedly a law of nature for the first-degreer - perhaps the most fundamental of them all. The first degree account is thus perfectly coherent, avoids the MPPP, has significant explanatory value, and there are no concerns about causal overdetermination.

The necessity in the second degree conception is contingent, so although it is no accident that the path followed through velocity-configuration space is extremised (as it is governed

[29]If world w's property network gives rise to least action principles, then, in virtue of these properties, it is metaphysically necessary that the least action principles hold at w. However there are many possible worlds where the property networks do not give rise to this effect

by a law), the instantiation of the higher-order fact playing the governing role ensuring this is contingent. There are many possible worlds in which $Arm(PLA)$ is not a law, so the curve is not only logically, but also metaphysically contingent. Furthermore, due to its governing nature, $Arm(PLA)$ also seems to serve us well in explanation. As we saw, the equations of motion turn out not be laws, but this is a welcome consequence for the second-degreer, not a worry.

If one makes the assumption that the curve must be metaphysically contingent, one rules out the possibility of a third-degree interpretation of the PLA. However, one can only justify the claim that the PLA presupposes many *logically* possible curves between any two points in the velocity-configuration space, and this is not in itself incompatible with the third-degree view. The third-degreer does not explain why the only metaphysically possible path is that which extremises action, and some might find this troublesome (the fact that the PLA holds supervenes on the fundamental dispositional properties, and on them alone, leaves just brute metaphysical necessity), but why should there be such an explanation? After all, the third-degree view is, in essence, simply the view that these properties are the source of all physical modality – they ground any regularities that may emerge, so to ask for any further explanation is unfair.

Despite playing a fundamental role in all our best physical theories, surprisingly little in contemporary metaphysics has been written on the nature of atemporal laws such as the PLA. In this paper, however, we have shown how the proponents of each of the three degrees of physical modality can deal with such principles, without diverging from the fundamental metaphysical doctrines they espouse in terms of the instantaneous laws we are so much more accustomed to.

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