Topics in foundations of probability theory 4 lectures for PhD and MSc students

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The aim of the lectures is twofold: (i) to make students familiar with some concepts of measure theoretic probability theory that go beyond the most elementary notions both in terms of mathematical precision and in content; (ii) to show how the more advanced notions can be used to handle some classical philosophical-foundational issues (especially some "paradoxes"). While the material is heavily mathematical and several important theorems will be stated, proofs of theorems will not be given. It is not expected that students attending the lectures will have read the suggested readings.

1 Lecture 1: Basic concepts of measure theoretic probability theory

Probability measure spaces; random variables and their distribution; definition and properties of L^p $(p = 1, 2, \infty)$ spaces of random variables. Linear functionals on L^p $(p = 1, 2, \infty)$ spaces, dual spaces of, and duality theorems about L^p $(p = 1, 2, \infty)$ spaces.

Reading: Relevant sections from [1] (especially section 19.)

2 Lecture 2: Conditionalization

Absolute continuity of probability measures, Radon-Nikodym theorem. Definition and basic properties of conditional expectations on L^p $(p = 1, 2, \infty)$ spaces. The problem of statistical inference. Conditional probability as a Bayesian solution of the problem of statistical inference. Jeffrey rule and Bayes' rule as special cases of conditioning in terms of conditional expectations. General properties of Bayesian conditioning viewed as a map in the dual space of L^p spaces.

Reading: Relevant parts of sections 32. and 34. in [1]; [5]

3 Lecture 3 and 4. Treating some paradoxes: Bertrand's paradox and the Borel Kolmogorov paradox

- Probability as pure mathematics, application of probability and interpretation of probability. The classical interpretation and the classical formulation of Bertrand's Paradox. Haar measure and General Bertrand's Paradox. Why Bertrand's paradox is not paradoxical but is felt so.
- The problem of conditioning with respect to probability zero events: The Borel-Kolmogorov Paradox. In what sense does Kolmogorov's solution of the Borel-Kolmogorov paradox in terms of conditional expectations solve the Borel-Kolmogorov paradox.

Reading: [3], [4], [2]

References

[1] P. Billingsley. *Probability and Measure.* John Wiley & Sons, New York, Chichester, Brisbane, Toronto, Singapore, Third edition, 1995.

[2] Z. Gyenis, G. Hofer-Szabó, and M. Rédei. Conditioning using conditional expectations: The Borel-Kolmogorov Paradox. *Synthese*, 194:2595–2630, 2017. DOI 10.1007/s11229-016-1070-8.

[3] Z. Gyenis and M. Rédei. Defusing Bertrand's Paradox. *The British Journal for the Philosophy of Science*, 66:349–373, 2015.

[4] Z. Gyenis and M. Rédei. Why Bertrand's Paradox is not paradoxical but is felt so. In U. Maki, S. Ruphy, G. Schurz, and I. Votsis, editors, *Recent Developments in the Philosophy of Science: EPSA13 Helsinki*, pages 265–276. Springer, 2015.

[5] Z. Gyenis and M. Rédei. General properties of Bayesian learning as statistical inference determined by conditional expectations. *The Review of Symbolic Logic*, 10:719–755, 2017. preprint: http://philsci-archive.pitt.edu/11632/.